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Empirical prediction of response spectra

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ABSTRACT

Response spectra of earthquake accelerograms, including smoothed averages of such spectra, are descriptions of ground motion and may be used as a basis for defining a design spectrum. Two methods of deriving response spectra for use in sesimic design are reviewed. These are: scaling dynamic amplification factors by peak ground motions and computing response spectral ordinates by means of attenuation relationships. It is shown that, if comparisons are made in a consistent manner, application of each of these methods gives similar spectra. However, the standard procedure of scaling dynamic amplification factors by peak ground motions can lead to biased estimates of response spectral ordinates. Reasons for this are related to the fact that spectral shape is dependent on earthquake magnitude and to the manner in which a scaled spectrum is computed.

INTRODUCTION

The response spectrum of a particular accelerogram is a highly irregular function of frequency or period. For purposes of describing ground motion, it is desireable to have a smooth function of frequency or period which envelops the irregular spectra below a particular level. Such a smooth spectrum is also useful for design, in which case it is a specification of the performance requirements of a structure.

Two procedures for the determination of smooth response spectra are

1) scale dynamic amplification factors by peak ground motions, and
2) compute spectra by means of an attenuation relationship.

Each of these methods is based on empirical data but on quite different assumptions. However, in the following it will be shown that, if comparisons are made in a consistent manner, the above procedures result in similar response spectra with differences that can be explained.

SCALED RESPONSE SPECTRA

A response spectrum can be divided into three different period ranges within which amplification of peak ground motion is relatively constant: the low period or 'acceleration' range, the intermediate period or 'velocity' range, and the long period or 'displacement' range. In each region the corresponding peak ground motion is amplified the most.

Dynamic amplification factors for peak ground motions have been derived by numerous inves-

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[I] Acres International Ltd., Niagara Falls, Ontario, Canada L2E 6W1 tigators using slightly different suites of accelerograms (Newmark and Hall, 1982 and references therein). The most recent are given in Newmark and Hall (1982) and reproduced in Table 1.

As originally suggested by Newmark *et al* (1973), these amplification factors are multiplied by peak ground motions resulting in acceleration, velocity and displacement spectral ordinates which can be plotted on four-way log paper to produce a smooth response spectrum. There are two variants of the original Newmark-Hall method of computing a scaled spectrum. These are given below.

Procedures for Computing a Scaled Spectrum

Four-way logarithmic construction

Procedure A

1) Obtain peak ground motions pga, pgv and pgd.
2) Plot pga, pgv and pgd, draw horizontal line through pgv, a line parallel to displacement axis through pga and a line parallel to acceleration axis through pgd. Connect all lines.
3) Obtain median or 84 percentile amplification factors, F_a, F_v and F_d from Table 1.
4) Multiply $pga \times F_a = A$, $pgv \times F_v = V$ and $pgd \times F_d = D$.
5) Plot A, V and D. Draw horizontal line through V, a line parallel to displacement axis through A and a line parallel to acceleration axis through D. Connect all lines.
6) Draw a line between the A ordinate at 0.125 sec period (8 Hz) and the pga line at 0.03 sec period (33 Hz).

Procedure B

1) Obtain peak ground acceleration pga.
2) Estimate pgv and pgd from published ratios pgv/pga, $pga \cdot pgd/pgv^2$ appropriate to site conditions.
3) Plot pga, pgv and pgd, draw horizontal line through pgv, a line parallel to displacement axis through pga and a line parallel to acceleration axis through pgd. Connect all lines.
4) Follow steps 3-6 of procedure A.

An important variable in each of these procedures is the percentile level of the amplification factors to be used. The use of 84 percentile amplification factors would result in a spectrum that represents uncertainty or variability in the ground motion. Accounting for ground motion variability may be desirable for purposes of ground motion description and is certainly necessary in some seismic design applications. However, care must be taken to ensure that the variability is not counted twice. For example, peak ground motions are often estimated by probabilistic or other procedures which account for ground motion variability. Using such estimates to scale 84 percentile amplification factors would be a case of 'compounding conservatisms' (Housner and Jennings, 1982; Donovan and Becker, 1986).

ATTENUATION RELATIONSHIPS FOR SPECTRAL ORDINATES

Attenuation relationships for spectral ordinates at a particular natural vibration period can be derived. This is a natural extension of the attenuation relationships used to compute peak ground acceleration given a magnitude and distance in that such relationships are for spectral response at low natural period. Several attenuation relationships for spectral ordinates exist and

TABLE 1

Spectral Amplification Factors

From Newmark and Hall (1982)

	84 percentile			Median		
Damping ratio (%)	F_a	F_v	F_d	F_a	F_v	F_d
0.5	5.10	3.84	3.04	3.68	2.59	2.01
1	4.38	3.38	2.73	3.21	2.31	1.82
2	3.66	2.92	2.42	2.74	2.03	1.63
3	3.24	2.64	2.24	2.46	1.86	1.52
5	2.71	2.30	2.01	2.12	1.65	1.39
7	2.36	2.08	1.85	1.89	1.51	1.29
10	1.99	1.84	1.69	1.64	1.37	1.20
20	1.26	1.37	1.38	1.17	1.08	1.01

TABLE 2

Spectral Attenuation Relationships

$$\log y = a + b(\mathrm{M}-6) + c(\mathrm{M}-6)^2 + d\log R + kR + s$$

y - randomly oriented horizontal component

After Joyner and Boore (1988)

T(sec)	a	b	c	d	h	k	s	σ
	Pseudo-velocity (cm/sec) 5% damping							
0.10	2.16	0.25	-0.06	-1.0	11.3	-0.0073	-0.02	0.28
0.15	2.40	0.30	-0.08	-1.0	10.8	-0.0067	-0.02	0.28
0.20	2.46	0.35	-0.09	-1.0	9.6	-0.0063	-0.01	0.28
0.30	2.47	0.42	-0.11	-1.0	6.9	-0.0058	0.04	0.28
0.40	2.44	0.47	-0.13	-1.0	5.7	-0.0054	0.10	0.31
0.50	2.41	0.52	-0.14	-1.0	5.1	-0.0051	0.14	0.33
0.75	2.34	0.60	-0 16	-1.0	4.8	-0.0045	0.23	0.33
1.0	2.28	0.67	-0.17	-1.0	4.7	-0.0039	0.27	0.33
1.5	2.19	0.74	-0.19	-1.0	4.7	-0.0026	0.31	0.33
2.0	2.12	0.79	-0.20	-1.0	4.7	-0.0015	0.32	0.33
3.0	2.02	0.85	-0.22	-0.98	4.7	0.0	0.32	0.33
4.0	1.96	0.88	-0.24	-0.95	4.7	0.0	0.29	0.33
	Peak acceleration (g)							
	0.43	0.23	0.0	-1.0	8.0	-0.0027	0.0	0.28
	Peak velocity (cm/sec)							
	2.09	0.49	0.0	-1.0	4.0	-0.0026	0.17	0.33

are reviewed in Joyner and Boore (1988).

As an example, Joyner and Boore (1988) assumed that a peak ground motion parameter or a response spectral ordinate, y, at a particular natural period could be predicted by a function of the form

$$\log y = a + b(\mathrm{M}-6) + c(\mathrm{M}-6)^2 + d\log R + kR + s \pm \epsilon. \tag{1}$$

$$s = \begin{cases} \neq 0 & \text{soil site} \geq 5\text{m thickness} \\ 0 & \text{rock site} \end{cases}$$

$$5 \leq \mathrm{M} \leq 7.7$$

$$R = (r^2 + h^2)^{\frac{1}{2}}$$

where M is moment magnitude, r is the distance to the vertical projection on the earth's surface of the nearest point of rupture, and h is a constant. Base 10 logarithms are used. The constants a, b, c, d, h, k and s are determined empirically using regression methods.

The term ϵ is the random error in $\log y$ which has a Gaussian or normal probability distribution with zero mean and standard deviation σ. Thus $\log y$ has a normal distribution or, equivalently, y is lognormally distributed having a median value, y_{50}, given by Equation 1 with $\epsilon = 0$. The median plus one standard deviation or 84 percentile value is given by $y_{84} = y_{50}10^{\sigma}$. The standard deviation is believed to be mainly a reflection of travel path and local site condition variability. Note that the reason for the existence of the error term in Equation 1 is the same as the reason for the median and 84 percentile amplification factors in Table 1.

The constants in Equation 1 were estimated using the spectral ordinates of the horizontal components of a selected suite of accelerograms from western North America. Coefficients for the randomly oriented horizontal component were estimated by considering both horizontal components at a recording site as independent data. These coefficients are shown in Table 2.

A COMPARISON

A response spectrum computed using the JB attenuation relationships can be meaningfully compared with NH response spectra since the empirical basis of both types of spectra is horizontal component accelerograms from western North America. Assuming that a M6.0 earthquake occurs at a distance of $r = 20$ km, the Joyner and Boore attenuation relationships for median values on a rock site ($s = 0$) result in pseudo-velocity ordinates which are converted to pseudo-acceleration (PSA) ordinates to give the response spectrum labelled JB shown in Figure 1.

For the same earthquake, the relationships for pga and pgv in Table 2 give the median values

$$pga = 0.109\text{g} \qquad pgv = 5.34 \text{ cm/sec.}$$

These are used to compute the Newmark-Hall spectrum for 5% damping using median amplification factors in procedure A. This results in the spectrum labelled NH-A shown in Figure 1. This linear plot of an NH spectrum was produced by directly plotting the value $PSA = A = pga \times F_a$ in the short period 'acceleration' range and converting $V = pgv \times F_v$ to PSA in the intermediate period 'velocity' range. It is possible that the long period (> 1 sec) portion of the linear plot is in the 'displacement' range of the spectrum.

Assuming the same value of pga and that the value of pgv is not given, procedure B can also be used to compute a spectrum. An empirical ratio pgv/pga for rock sites is given in Newmark and Hall (1982)

$$pgv/pga = 91.4 \text{ cm/sec/g.}$$

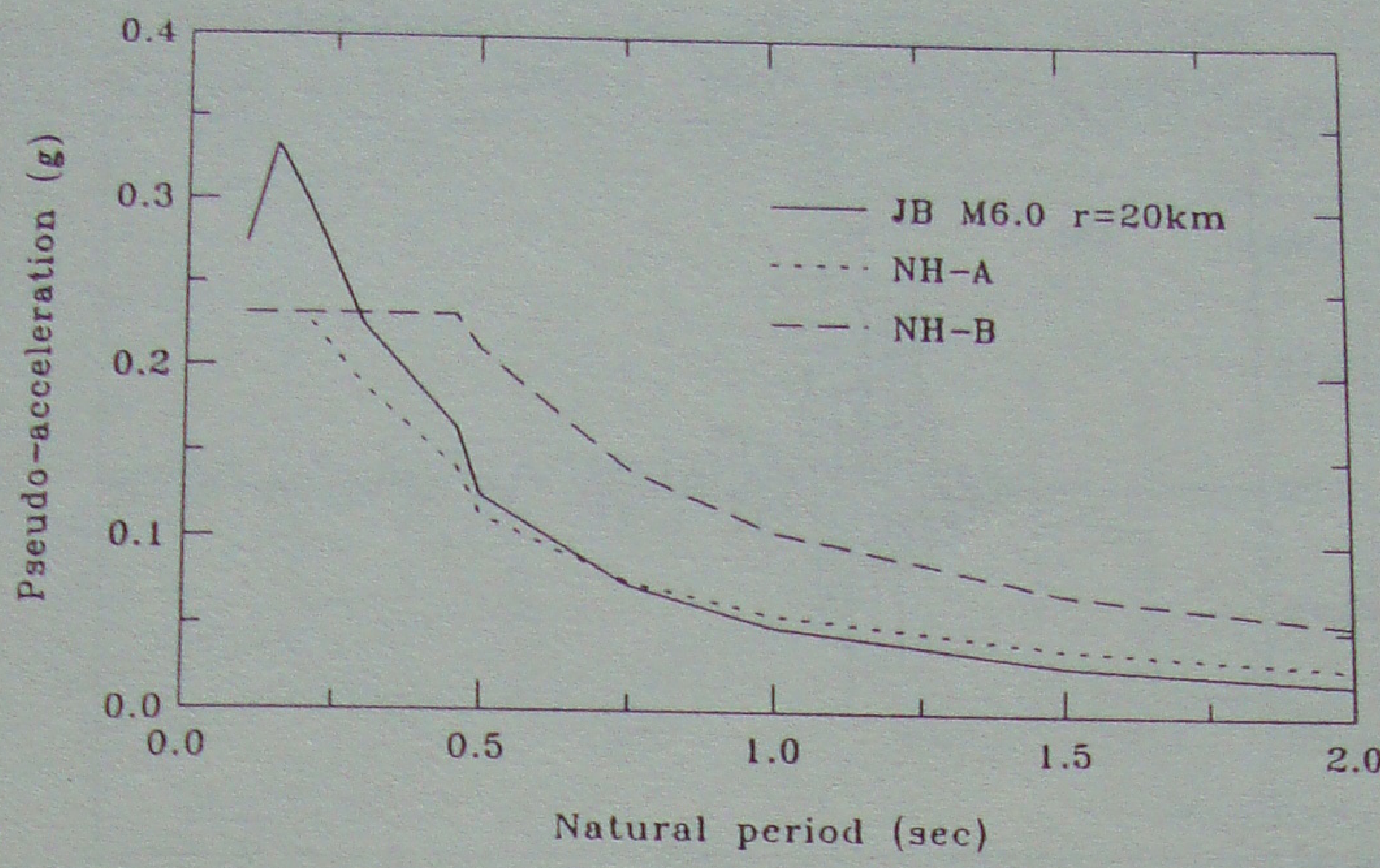

Figure 1. Comparison between Joyner and Boore spectrum and Newmark-Hall spectra computed by procedures A and B.

Given *pga* = 0.109g, the above ratio gives *pgv* = 9.97 cm/sec. This results in the spectrum labelled NH-B in Figure 1.

Despite the differences in their derivation, the JB and NH-A spectra are remarkably similar for periods greater than 0.5 sec. At periods less than about 0.3 sec, there is a significant difference between JB and both NH spectra. This is partly due to the assumption of constant pseudo-acceleration in this period range by procedures A and B. However, NH-B exhibits larger amplification than either JB or NH-A for periods greater than 0.3 sec. The likely reasons for these discrepancies are discussed in the next section.

DEPENDENCE OF SPECTRAL SHAPE ON MAGNITUDE

The amount of low frequency energy in ground motion generated by large magnitude earthquakes is relatively larger than that generated by small earthquakes. Figure 2a shows the accelerogram due to a M6.2 earthquake recorded by an accelerometer of the SMART array in Taiwan. Note the 1 Hz and lower frequencies in this ground motion. A M4.9 aftershock of this earthquake occurred and was recorded by the same accelerometer. The resulting accelerogram is shown in Figure 2b. The high frequency (> 1 Hz) in the ground motion is evident. Both these accelerograms were recorded at almost the same distance from the earthquake and at the same site, so that no complicating factors are present in this comparison.

Studies showing similar dependence of spectral shape on magnitude are described in Trifunac and Anderson (1978) and Iwasaki (1981).

There exists the possibility that the Newmark-Hall spectrum is biased toward that of a large magnitude earthquake. The 28 accelerograms used by Newmark *et al* (1973) to derive amplification factors were recorded during nine earthquakes, seven of which were greater than M6.0. The distribution of magnitudes of these events is shown in the bar diagram of Figure 3a. The preponderance of accelerograms due to earthquakes greater than M6.0 is evident. It should be noted that there were few accelerograms due to earthquakes of magnitude M< 6 at the time the amplification factors were derived.

Joyner and Boore used 64 horizontal component accelerograms recorded during 12 events in

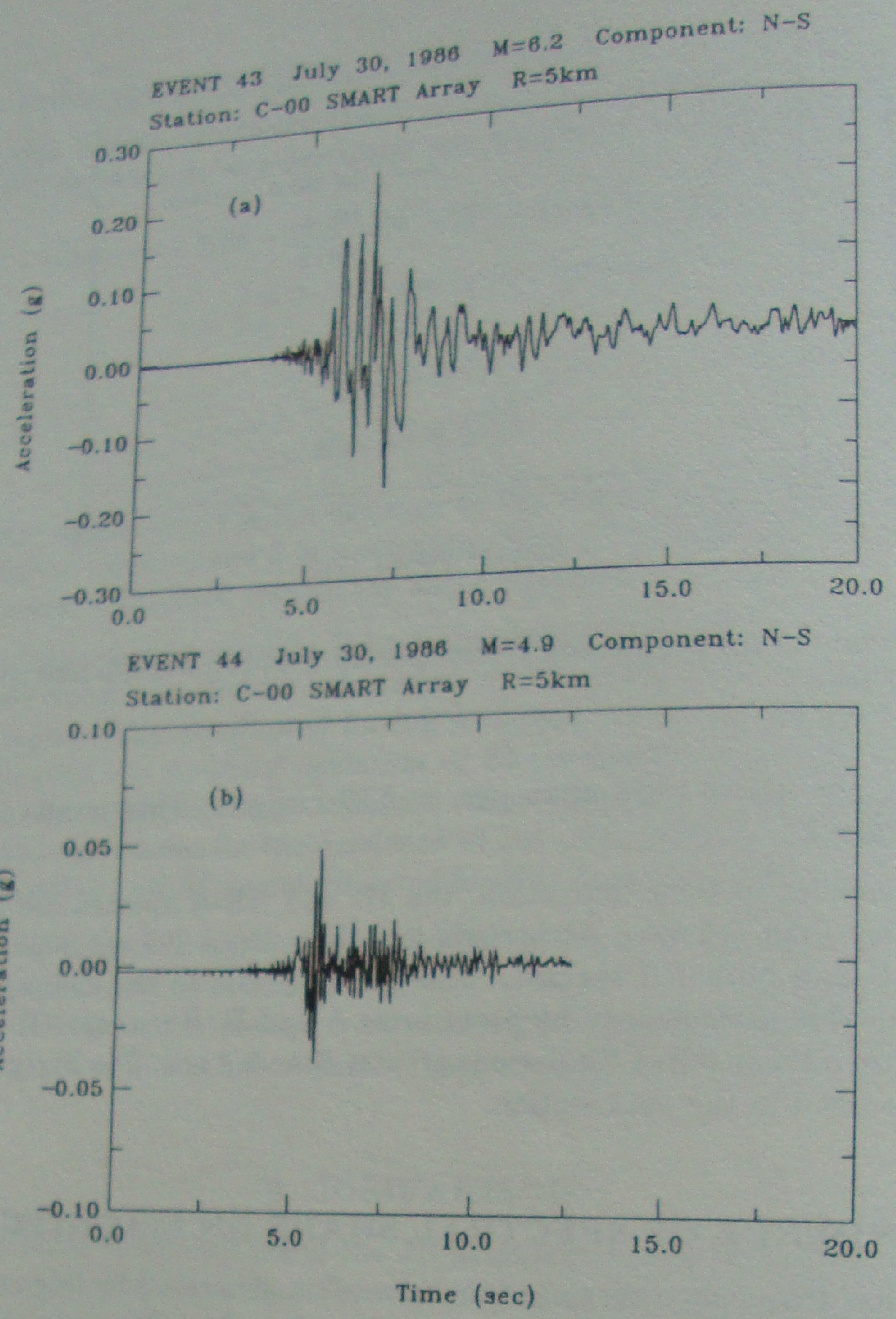

Figure 2. Accelerograms of M6.2 and M4.9 earthquakes recorded by SMART array in Taiwan.

their study. The distribution of magnitudes of these events is shown in the bar diagram of Figure 3b. It may be seen that the magnitudes of the events selected by Joyner and Boore are more evenly distributed in the range $5.0 \leq M \leq 8.0$.

Since the bias in the Newmark-Hall spectrum is evidently toward large magnitudes, it would explain the lower amplification of Newmark-Hall spectra in the low period range, as shown in Figure 1. However, at longer periods, the possible bias of the Newmark-Hall spectra toward larger values may not be significant.

Bias due to magnitude dependence can also be introduced by estimating values of *pgv* and *pgd* using published ratios between these ground motion parameters and *pga*. Figure 4 shows a plot of the ratio *pgv*/*pga* versus magnitude. This was also computed using the JB attenuation relationships for rock. From this plot it may be seen that the ratio *pgv*/*pga* is a strong function of magnitude. The dependence of *pgv*/*pga* on magnitude was also noticed in the seismic zoning maps of Canada by Heidebrecht *et al* (1983). Using the same data base as that used by Joyner and Boore (1988), Donovan and Becker (1986) empirically derived a relationship for the ratio *pgv*/*pga* which suggests the same strong magnitude dependence of this ratio. Thus, the practice

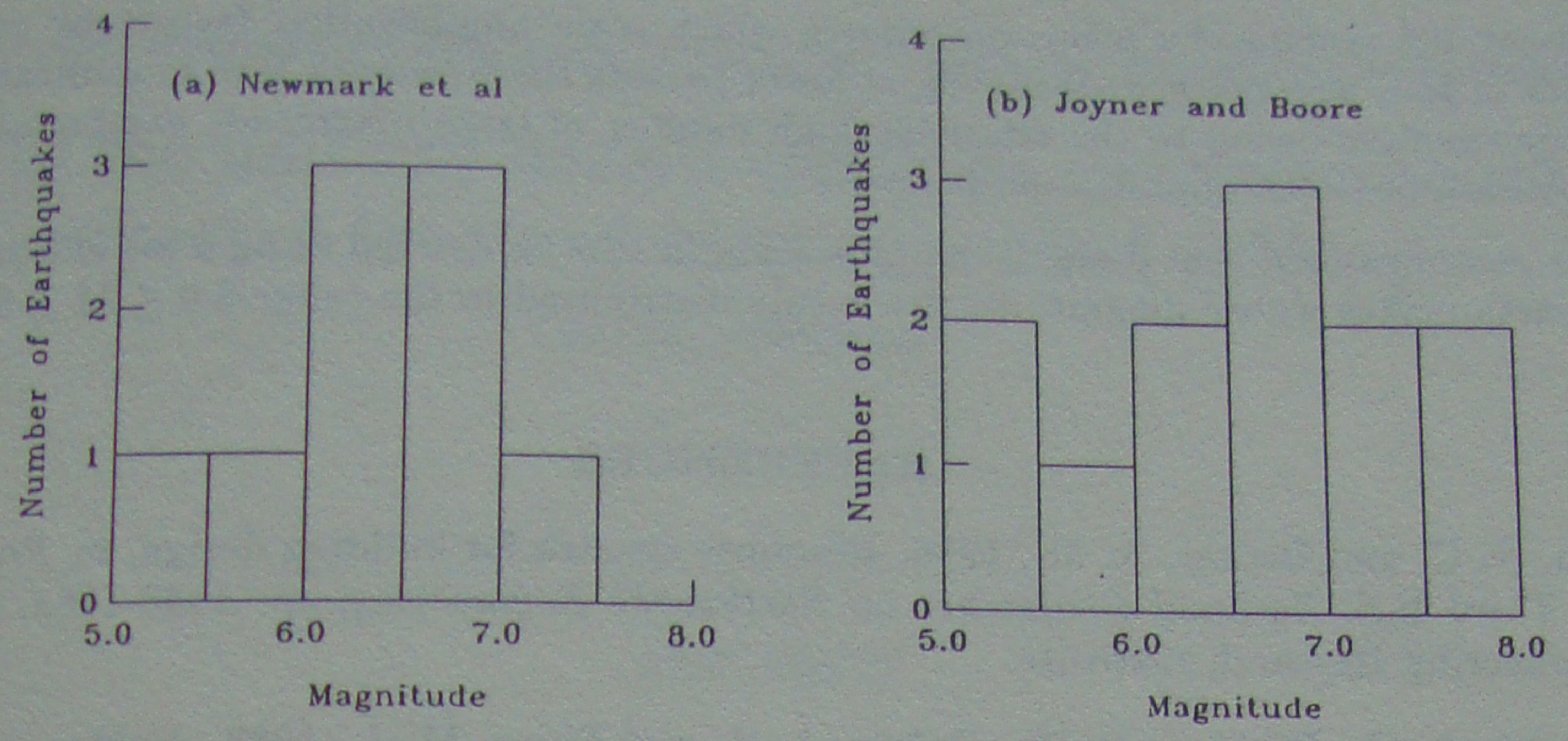

Figure 3. (a) Distribution of earthquake magnitudes used by Newmark *et al* (1973) to derive amplification factors. (b) Distribution of earthquake magnitudes used by Joyner and Boore (1988) to derive attenuation relationships for spectral ordinates.

of estimating *pgv* from published *pgv/pga* ratios can lead to significant magnitude bias in the resulting scaled spectrum. A similar conclusion likely applies to estimating *pgd* from the ratio $pga \cdot pgd/pgv^2$.

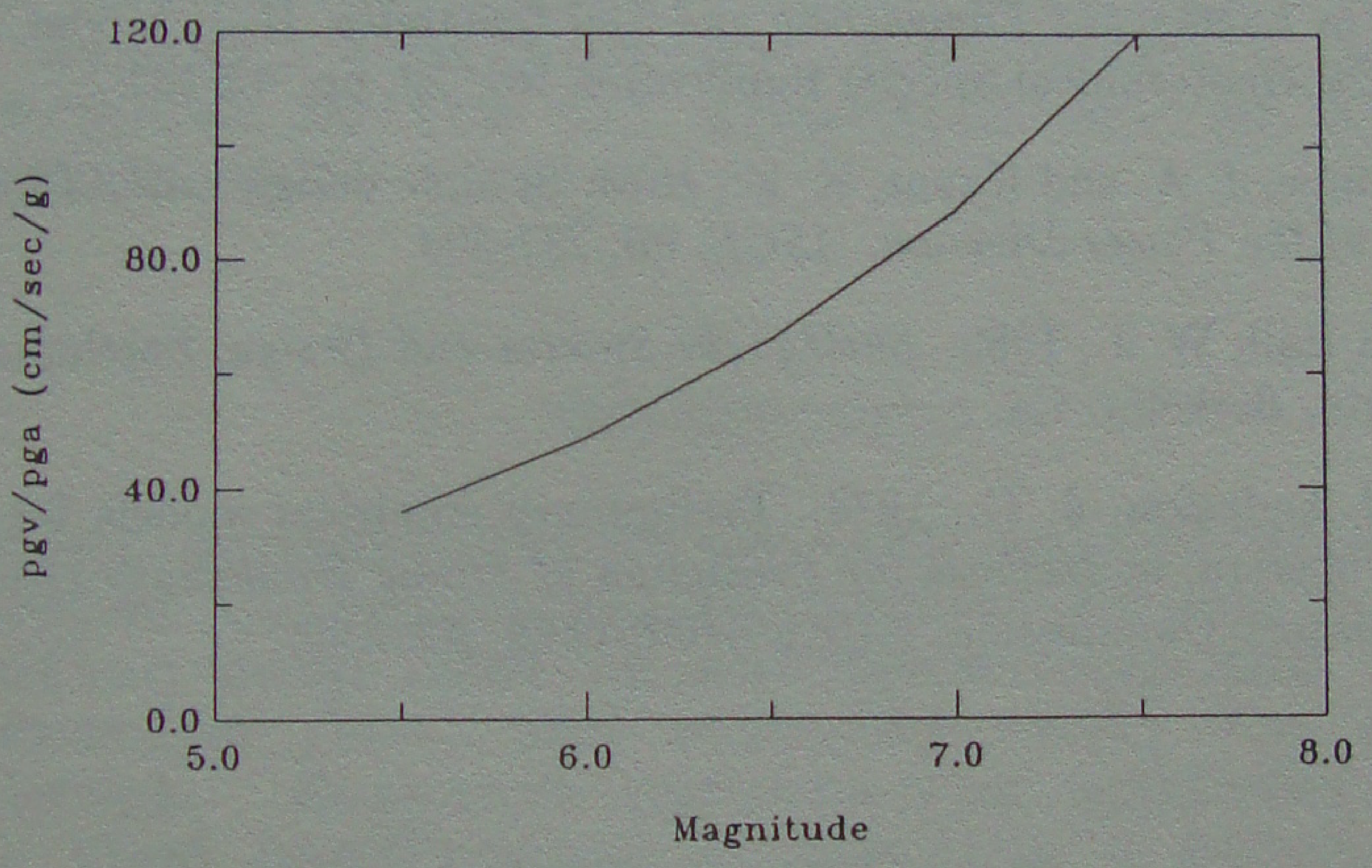

Figure 4. Plot of the ratio *pgv/pga* versus earthquake magnitude. This curve was computed using $r = 20$km in the Joyner and Boore attenuation relationships for rock given in Table 2.

CONCLUSIONS AND RECOMMENDATIONS

Since the shape of a response spectrum depends on earthquake magnitude, the procedure of scaling dynamic amplification factors by peak ground motions to obtain a smooth response spectrum can lead to biased response spectra. The bias can be reduced at natural periods greater than about 0.3 seconds by using procedures which scale amplification factors by independent estimates of *pga*, *pgv* and *pgd*. However, at lower periods the Newmark-Hall amplification factors are unconservative owing to the relatively high number of large magnitude earthquakes used in the derivation of these amplification factors.

It is recommended that dynamic amplification factors be derived using a set of accelerograms due to earthquakes whose magnitudes are evenly distributed in the range $5.0 \leq M \leq 8.0$.

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